Quiz 4 (10pts)
Math 214 Section Q1 Winter 2010

Your name:_____ ID#:_____

Please, use the reverse side if needed.

1.(5 pts) Find the Taylor series for the function $f(x) = e^{2x}$ at $x = 1$.

Solution.

The nth derivative of f equals

$$f^{(n)}(x) = 2^n e^{2x}.$$
$$f^{(n)}(1) = 2^n e^2.$$

Therefore, the Taylor series for $f(x)$ at $x = 1$ is

$$\sum_{n=0}^{\infty} \frac{2^n e^2}{n!}(x-1)^n.$$

2.(5 pts) Use series to evaluate the limit

$$\lim_{x\to 0} \frac{\sin x - x - x^3}{x^3}$$

Solution. We will use the Maclaurin series for $\sin x$.

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

Then

$$\lim_{x\to 0} \frac{\sin x - x - x^3}{x^3} = \lim_{x\to 0} \frac{(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots) - x - x^3}{x^3}$$
$$= \lim_{x\to 0} \frac{-\frac{7x^3}{6} + \frac{x^5}{5!} - \cdots}{x^3} = \lim_{x\to 0} \left(-\frac{7}{6} + \frac{x^2}{5!} - \cdots\right) = -\frac{7}{6}.$$